

1.

$$a) 4^{-2x+1} = 8^{x-4}$$

$$(2^2)^{-2x+1} = (2^3)^{x-4}$$

$$2^{-4x+2} = 2^{3x-12}$$

$$-4x+2 = 3x-12$$

$$14 = 7x$$

$$\rightarrow x = 2$$

$$\text{check: } \text{left: } 4^{-(2 \cdot 2)+1} = 4^{-3} = \frac{1}{64}$$

$$\text{Right: } 8^{2-4} = 8^{-2} = \frac{1}{64} \quad \checkmark$$

$$(b) 9^{2x+1} = \left(\frac{1}{27}\right)^{x+2}$$

$$(3^2)^{2x+1} = (3^{-3})^{x+2}$$

$$3^{4x+2} = 3^{-3x-6}$$

$$4x+2 = -3x-6$$

$$\rightarrow 7x = -8$$

$$x = -8/7$$

check: w/ calculator

$$\text{Left: } 9^{2(-8/7)+1} \approx 0.0593$$

$$\text{Right: } \left(\frac{1}{27}\right)^{(-8/7)+2} \approx 0.0593$$

OK

$$(c) 4^{x^2-x} = 1$$

$$x^2-x = 0$$

$$x = 0 \text{ or } x = 1$$

Check

$$x=0: 4^{0^2-0} = 4^0 = 1 \quad \checkmark$$

$$x=1: 4^{1^2-1} = 4^0 = 1 \quad \checkmark$$

1 d) $\log_{3x} 36 = 2$

$$(3x)^2 = 36$$

$$\textcircled{3x = 6} \text{ or } \underline{3x = -6}$$

not allowed, because < 0

check: $\log_6 36 = 2$ ✓

#2 $\log_6 58 = \frac{\log_{10}(58)}{\log_{10}(6)} \approx \frac{1.7634}{0.77815} \approx \underline{2.2662}$

3 a) $0.2y = 5(7)^x$

$$\frac{1}{5}y = 5(7)^x$$

$$\frac{y}{25} = 7^x$$

$$\underline{x = \log_7 (y/25)}$$

3 b) $7^{2x-1} = 17^x$

$$\underline{\log_{17}(7^{2x-1}) = 1x}$$

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4a) $\log_3(y/5) = x$

$$y/5 = 3^x$$

$$\underline{y = 5 \cdot 3^x}$$

4b) $\log_{10}(0.5y) = x$

$$0.5y = 10^x$$

$$\underline{y = 2 \cdot 10^x}$$

#59 $6^{5x+1} = 4^x$

$$4^x = 6^{\log_6(4^x)}$$

substitute $\leftarrow$

$$6^{5x+1} = 6^{x \log_6(4)}$$

$$5x+1 = x \cdot \log_6(4)$$

$$(5 - \log_6(4))x + 1 = 0$$

$$x = \frac{1}{\log_6(4) - 5} \approx 0.2366139$$

≈ 0.24 to the nearest 100th

#56 $7.5^x = 2^{x+1}$

$$7.5^x = 5^{\log_5(2^{x+1})}$$

$$7.5^x = 5^{(x+1) \log_5(2)}$$

$$\log_5(7) + x = (x+1) \log_5(2)$$

$$x = \frac{\log_5(7) - \log_5(2)}{\log_5(2) - 1} \approx -0.284107$$

$x \approx -0.28$ to the nearest 100th

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$$\begin{aligned} \#6 \\ (a) \quad & \log_5(10) + \log_5(75) - (\log_5(2) + \log_5(3)) \\ &= \log_5\left(\frac{10 \cdot 75}{2 \cdot 3}\right) = \log_5(125) = \underline{\underline{3}} \end{aligned}$$

$$\begin{aligned} (b) \quad & \log_5(625^{1/5}) = 1.5 \log_5(625) \\ &= \frac{3}{2} \cdot 4 = \underline{\underline{6}} \end{aligned}$$

$$\begin{aligned} \#7 (a) \quad & 5 \log(x) - \frac{1}{2} \log(y) - 3 \log(z) \\ &= \log(x^5) - \log(y^{1/2}) - \log(z^3) \\ &= \log\left(\frac{x^5}{y^{1/2} z^3}\right) \\ &= \underline{\underline{\log\left(\frac{x^5}{y^{1/2} z^3}\right)}} \end{aligned}$$

$$\begin{aligned} (b) \quad & \frac{3}{4} \log_a(b) + 4 \log_a(c) - 3 \log_a(b) \\ &= \log_a(b^{3/4}) + \log_a(c^4) - \log_a(b^3) \\ &= \log_a\left(\frac{b^{3/4} c^4}{b^3}\right) = \underline{\underline{\log_a(c^4 / b^{9/4})}} \end{aligned}$$

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8a $\log_5(x) - \log_5(x-1) = \log_5(3)$

$$\log_5\left(\frac{x}{x-1}\right) = \log_5(3)$$

$$\frac{x}{x-1} = 3$$

$$x = 3/2$$

8b $\log_4(x-5) + \log_4(x-2) = 1$

$$\log_4((x-5)(x-2)) = 1$$

$$(x-5)(x-2) = 4$$

$$x = 5$$

$$\text{or } x = 2$$

8c $\log(x) = \frac{2}{3} \log(27) + 2 \log(2) - \log(3)$

$$\log(x) = \log\left(\frac{2 \cdot 7^{2/3} \cdot 2^2}{3}\right)$$

$$\log(x) = \log(12)$$

$$x = 12$$

8d $2 \log_3(x) - \log_3(2x) = 2$

$$\log_3\left(\frac{x^2}{2x}\right) = 2$$

$$\log_3\left(\frac{x}{2}\right) = 2$$

$$\Rightarrow \frac{x}{2} = 3^2$$

$$x = 18$$

#9

$$\log(3) = a, \quad \log(8) = b$$

$$\uparrow \text{ means } \log(2) = b/3$$

$$18 = 2 \cdot 3 \cdot 3$$

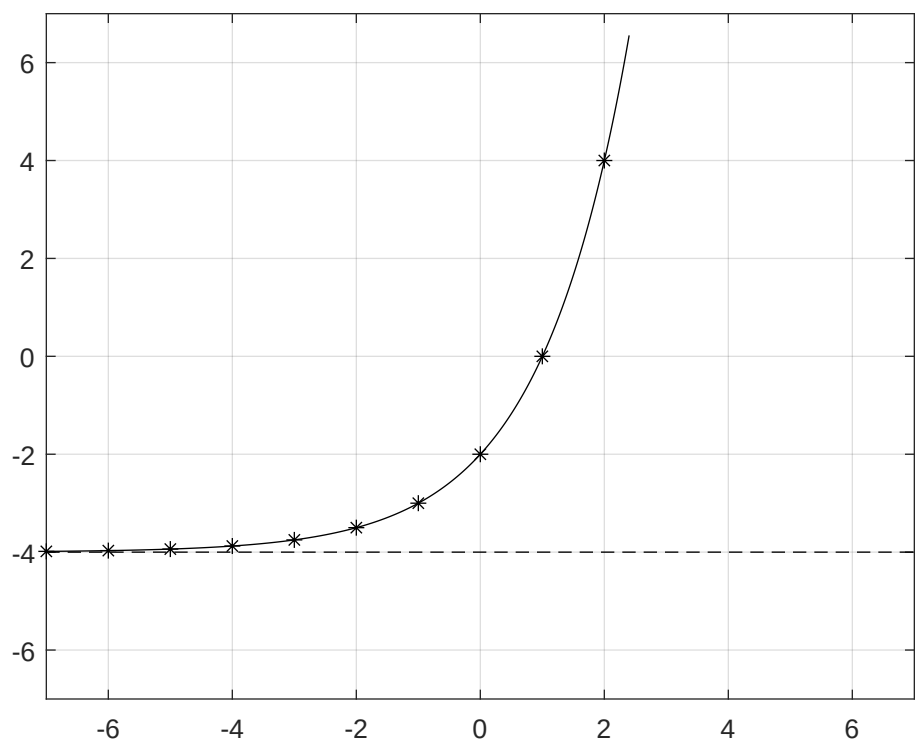
$$\log(18) = \log(2 \cdot 3 \cdot 3)$$

$$\log(18) = \log(2) + \log(3) + \log(3)$$

$$\log(18) = a + \frac{2}{3}b$$

#10 coord's

x	y	point
0	$2^0 - 4 = -2$	(0, -2)
1	$2^1 - 4 = 0$	(1, 0)
2	$2^2 - 4 = 4$	(2, 4)
-1	$2^{-1} - 4 = -3.5$	(-1, -3.5)
-2	$2^{-2} - 4 = -3.75$	(-2, -3.75)



#11

IF 800 is invested at 7.5% compounded annually

$$2000 = 800(1 + 0.075)^x$$

$$\frac{5}{2} = 1.075^x$$

$$\ln(5/2) = x \ln(1.075)$$

$$x = \frac{\ln(5/2)}{\ln(1.075)}$$

$$x \approx 12.67$$

#12

Hive increases by 35% in 4 months

means

$$1.35 = e^{4r}$$

if we measure time
in months

$$r = \frac{\ln(1.35)}{4}$$

Find the rate r .

IF we want 10,000 at $t = 18$, this means

$$10,000 = B_0 e^{18r}$$

$$B_0 = 10,000 e^{-18r} = \underline{2591.18}$$

Start with 2591 bees

(p 9)

#13 The amount of Carbon 14 at time t is $A(t) = A_0 e^{-rt}$ where A_0 is the initial amount.

so $60 = 100 e^{-rt}$, and t is the answer.

we need the rate r .

Since the $\frac{1}{2}$ life of carbon 14 is 5700 years,

$$\frac{1}{2} = e^{-5700r}$$

$$r = \frac{\ln(2)}{5700}$$

so $60 = 100 e^{-\left(\frac{\ln(2)}{5700}\right)t}$

$$t = \frac{5700 \ln(5/3)}{\ln(2)} \approx 4200 \text{ years}$$

#14

(p.10)

Major
magnitude 8.7 means the energy
of that quake is $10^{8.7}$

Let x be the magnitude of a minor quake

$$10^{8.7} = 150 \cdot 10^x$$

$$10^{8.7} = (10^{\log_{10}(150)}) \cdot 10^x$$

$$10^{8.7} = 10^{x + \log_{10}(150)}$$

$$8.7 = x + \log_{10}(150)$$

$$x = 8.7 - \log_{10}(150)$$

$$x = 6.5 \text{ to the nearest } \frac{1}{10} \text{th}$$